Geographical effects on cascading breakdowns of scale-free networks

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Cascading breakdowns of real networks have resulted in severe accidents in recent years. In this paper, we study the effects of geographical structure on the cascading phenomena of load-carrying scale-free networks. Our essential finding is that when networks are more geographically constrained, i.e., more locally interconnected, they tend to have larger cascading breakdowns. Explanations are provided in terms of the effects of cycles and the distributions of betweenness over degrees.

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Recently, dynamical processes on networks have been highly concerned and widely investigated [1–3]. Among many of the dynamical features of networks, robustness has attracted much attention [4–7], with a major focus on scale-free (SF) networks, i.e., the degrees of nodes satisfy a power law distribution: $P(k) \sim k^{-\lambda}$, for their ubiquity in real systems [8]. The heterogeneity of the degrees often makes the scale-free networks sensitive to intentional attacks [6–9], while it is resilient to random breakdowns [5,6] and is resilient under cascading breakdowns by the reservoir effect of the hubs, which can sustain large amounts of grains [10]. However, for cascading failures, a SF network is fragile even if one attacks only one or a few nodes with the largest degrees or highest loads [11].

Since many real networks exist in two- or three-dimensional (3D) physical spaces, it is natural to study the geographical complex networks, and it has attracted much attention recently [12–16]. Recent studies have shown that geographical structures have significant influence on percolation thresholds [15]. Because many real systems—e.g., power grid networks, traffic lines, Internet—are sensitive to cascading failures and are located on the two-dimensional global surface, the influence of geographical structures on cascading breakdowns is of high importance, but up to now, it is rarely studied.

For different kinds of networks, the influence of the underlying geography varies. For example, a city's traffic line network relies much on the geography, because the load being carried, namely, the traffic, can be distributed only locally in view of geography. However, some other networks, such as the power network and the Internet rely less on the geography since these loads, specifically, the power or the data packets, can be distributed somewhat nonlocally. To account for this difference of influence, we study the cascading phenomena on a weighted lattice embedded SF (WLESF) network [16], in which a parameter A controls the influence of the geography on the network structures, ultimately, on the cascading phenomena. Each node in the network is supposed to carry a certain type of load, such as power or traffic; and if the node is broken down, its load will be redistributed to its network "neighbors." We investigate the Bak-Tang-Wiesenfeld (BTW) sandpile model [10,17] as a prototypical model of cascading breakdowns on the WLESF network, and we further study the betweenness distribution. Both of our studies validate that when the network is more loosely connected in the geographical view, i.e., when its connections are less local, it will be more robust under cascading failures as the networks will encounter fewer huge avalanche events.

The network is generated as follows [16]. It begins with an $L \times L$ lattice, with periodical boundary conditions, and for each node assigned a degree k drawn from the prescribed SF degree distribution $P(k) \sim k^{-\lambda}$, $k \geqslant m$. Then a node i is picked out randomly; according to a Gaussian weight function $f_i(r) = De^{-(r/A\sqrt{k_i})^2}$, it selects other nodes and establishes connections until its degree quota k_i is filled or until it has tried many enough times. Duplicate connections are avoided. The process is carried out for all the nodes in the lattice. The clustering parameter A controls the spatial density of the connections. For the large A limit, e.g., $A\sqrt{m} \gg L$, the weight function will be trivial, and the network becomes a SF random (SFR) network, i.e., random in network connections [18]. To compare, we also investigate lattice embedded SF (LESF) networks with nearest neighbor connections [12]. Here, we assume that the time scales governing the dynamics are much smaller than those characterizing the network evolvement. Thus, the static geographical network models are suitable for discussing the problem under investigation.

The rules we adopted for sandpile dynamics are as follows: (i) At each time step, a grain is added at a randomly chosen node i (such as fluctuations of the real loads, e.g., power of a station). (ii) If the height at node i reaches or exceeds a prescribed threshold z_i (e.g., the rated load), here we choose that z_i equals i's degree k_i; then it becomes unstable and the grains at the node topple to its adjacent nodes: $h_i = h_i - k_i$; and for each i's neighbor j: $h_j = h_j + 1$ (e.g., the breakdowns of a certain power station and the redistribution of the powers); during the transfer, there is a small fraction f of grains being lost, which plays the role of sinks without which the system becomes overloaded in the end (e.g., some power-consuming units may break down and will not need

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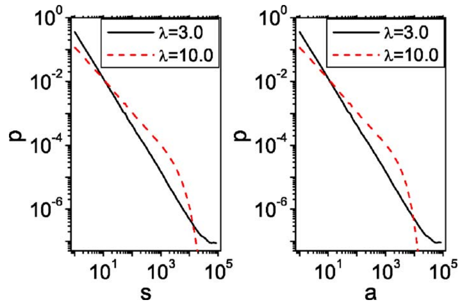

FIG. 1. (Color online) The probability distribution of avalanches for LESF networks out of 10^6 avalanche events on one network configuration. Left panel, probability distribution of avalanche size s; right panel, probability distribution of avalanche area a. The losing probability is $f=0.001$, and $m=4$, $N=10^5$. The data are log-binned.

power supply any longer). (iii) If this toppling causes any of the adjacent nodes to become unstable, subsequent topplings follow on those nodes in parallel until there is no unstable node left, forming an avalanche event (the cascadings). (iv) Repeat (i)–(iii).

There are many other models of cascading breakdowns on complex networks in the literature. For example, the fiber bundle model considers that the load is continuous and the driving is uniform, and the threshold or capacity of the each node is random with a given distribution [19]. There are also models that assume the threshold is proportional to the betweenness centrality of the node [11,20]. These models are similar to the model we discussed above since for most cases there is a relation between betweenness centrality b and the degree k: $b \sim k^{-\alpha}$ and α falls between 1 and 2 [9,21].

The main feature of the BTW sandpile model on the Euclidean space is the emergence of a power law with an exponential cutoff in the avalanche size distribution, $p(s) \sim s^{-\tau'} e^{-s/s_c}$, where s is the avalanche size, i.e., the number of toppling nodes in an avalanche event, and if a node toppled twice, it contributes 2 to the avalanche size; s_c is its characteristic size. In our studies, nodes toppled more than once in an avalanche event are seldom [10], except for the very large avalanches, which have already exceeded the exponential cutoffs. Thus we study the avalanche area a—the number of distinct nodes that toppled in an avalanche event—instead of avalanche size. The avalanche area distribution follows the same form as that of avalanche sizes:

$$p(a) \sim a^{-\tau} e^{-a/a_c}, \qquad (1)$$

where a_c is the characteristic area. A typical example is shown in Fig. 1.

For the BTW sandpile model on SFR networks, Goh *et al.* [10] have shown that the avalanche area exponent τ increases as λ decreases, caused by the increasing number of hubs playing the role of reservoirs, i.e., in the event of avalanches, the hubs may absorb many grains without topplings. Here, we will demonstrate that for the densely connected scale-free geographical networks, the reservoir effect is weakened, and the network has a smaller τ.

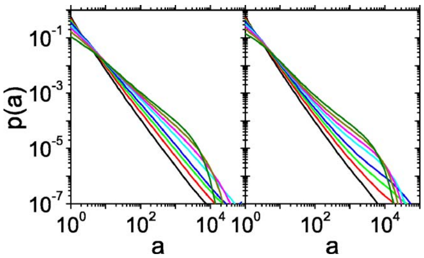

FIG. 2. (Color online) Avalanche area distribution for LESF (left panel) and WLESF $A=1$ (right panel) networks. For both panels, from up to down λ=10.0, 5.0, 4.0, 3.5, 3.0, 2.8, 2.6, and 2.4. The losing probability is $f=0.001$, and $m=4$,$N=10^5$. Ten network realizations are carried out and for each 10^6 avalanche events are recorded for statistics. The data are log-binned.

Figure 2 represents the avalanche area distribution for different λ of LESF networks and WLESF networks with $A=1$. It shows that as λ decreases, the curve of the avalanche area distribution is steeper, corresponding to larger τ. The trend is the same as the results in Ref. [10], indicating that the effect of hubs as reservoirs still exists. The avalanche area exponent τ for these data are fitted by formula 1 and presented in Fig. 3, together with that of SFR networks for comparison. The data for SFR networks we obtained is consistent with that of Ref. [10]. For large λ and large N limits, the SFR network tends to random graphs, for which $\tau \simeq 1.5$ [10,22], while the LESF network tends to a superlattice, with each node having m neighbors. Since in our studies $m=4$, the network limits to a normal 2D lattice, our result is $\tau=1.01(2)$, which is consistent with the previous results [17,23].

The avalanche area exponent for different A of WLESF network is shown in Fig. 4. As A becomes larger, the avalanche area exponent τ increases, and the curves of the avalanche area distribution become sharper in the double-log plot (see inset of Fig. 4), which corresponds to fewer large avalanche events. This transition in τ illuminates that when the network is geographically more loosely connected, it will be harder for large cascading events to occur.

The range of an edge is the length of the shortest paths

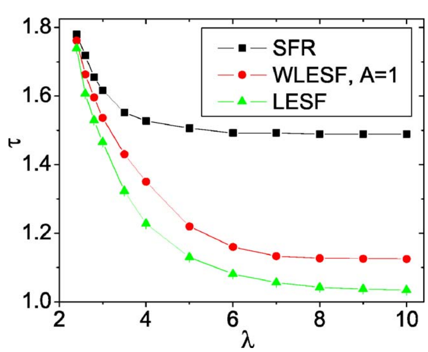

FIG. 3. (Color online) Avalanche area exponent τ vs the SF degree exponent λ, note that the errorbars in most cases are smaller than the symbol size. The data are fitted by formula 1, from the data presented in Fig. 2 and that of SFR networks. The network parameters and the statistics for SFR network are the same as that in Fig. 2.

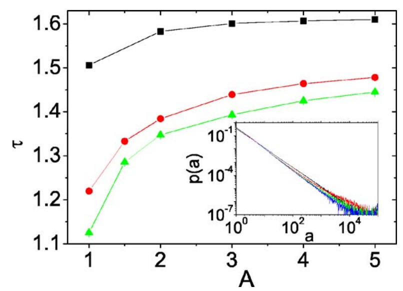

FIG. 4. (Color online) Avalanche area exponent τ vs the clusterness parameter A, for $\lambda=3.0$ (squares), 5.0 (circles), and 10.0 (triangles); note that the errorbars in most cases are smaller than the symbol size. The data are fitted by formula 1. Inset: avalanche area distribution for $\lambda=3.0$, from top to bottom are LESF, WLESF $A=1$, $A=2$, and SFR networks. Dynamical and network parameters are the same as that in Fig. 2.

between the nodes it connected in the absence of itself [2,24]. If an edge's range is l, the shortest cycle it lies on is of length $l+1$. Thus the distribution of range in a network sketches the distribution of shortest cycles. The inset of Fig. 5 shows that when the spatial constraint is slighter, as A goes larger, the range distribution drifts to larger ranges. It means that networks with loose spatial connections have fewer small order cycles but more higher order cycles. If there are many small order cycles, the toppling grains are more likely to meet, and the nodes with fewer grains, i.e., fewer than $z-1$, especially those with $z-2$ or $z-3$ grains, could also reach the toppling threshold z and topple. For example, let $ABCD$ be a quadrangle, and A, B, and D are all in their critical height z_A-1, z_B-1, and z_D-1, respectively; if A topples, then B and D will also topple cascadingly, thus C will receive 2 grains. So even if C has less grains than its critical height z_C-1, it could also topple. Larger order cycles contribute less to this effect. The main frame of Fig. 5 shows

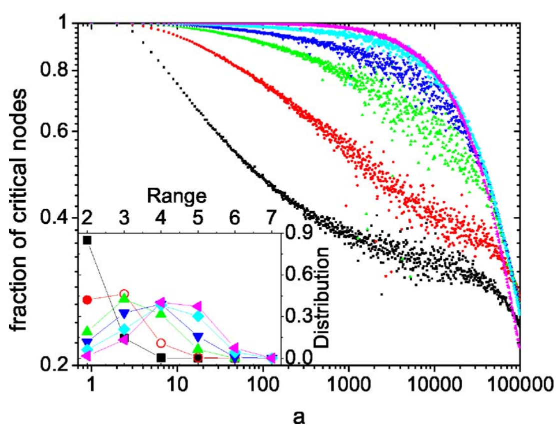

FIG. 5. (Color online) Fraction of nodes that toppled after receiving only one grain in an avalanche event vs avalanche area. From bottom to top is LESF (squares), WLESF $A=1$ (circles), $A=2$ (up triangles), $A=3$ (down triangles), $A=5$ (diamonds), and SFR network (left triangles). Each has 10^6 avalanche records on one network for statistics. $\lambda=3$, $m=4$, $N=10^5$. The losing probability is $f=0.001$. Inset: range distribution of the same networks; same symbols represent same networks as that in the main frame.

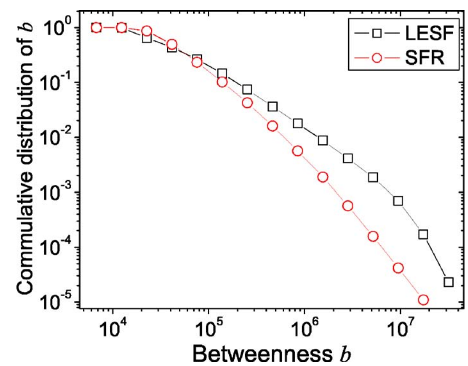

FIG. 6. (Color online) Log-binned cumulative distribution of betweenness b of the networks. $\lambda=4.0$, $m=2$, and network size $N=10^4$, each has been averaged over 100 configurations.

the fraction of nodes toppled in avalanches that have precisely $z-1$ grains. As the network is less geographically constrained and has fewer small order cycles, the fraction of toppling nodes with $z-1$ grains increases, substantiating our reasoning. This effect contributes to the large avalanche events of the densely connected networks and explains the decrease of the avalanche area exponent τ as the network is more geographically constrained.

In the following section, we studied the betweenness distribution of these geographical networks. The betweenness, or betweenness centrality, of node i is defined as the total number of shortest paths between pairs of nodes that pass through i; if a pair of nodes has two shortest paths, the nodes along those paths are given a betweenness of $\frac{1}{2}$ each [9,25]. The betweenness distribution for SF networks is reported to follow a power law $P_B \sim b^{-\delta}$, and for $2<\lambda\leqslant 3$, the exponent is $\delta\approx 2.2(1)$ [26]. We find that the betweenness distribution of the LESF network decays much slower than that of SFR networks, as Fig. 6 demonstrates for a particular case. The distributions for WLESF networks lay between them but do not appear in the graph for the purpose of clarity. The same holds for other λ and m values. Thus, there are more large betweenness nodes in LESF networks than in SFR networks. To help comprehending this, we present the density plot of the betweenness vs node's degree in Fig. 7 [9,21]. For LESFnetworks the betweenness of the same degree is distributed much more diffusively and, on average, is larger. It can be seen that even nodes with small degree k can have unusually large betweenness.

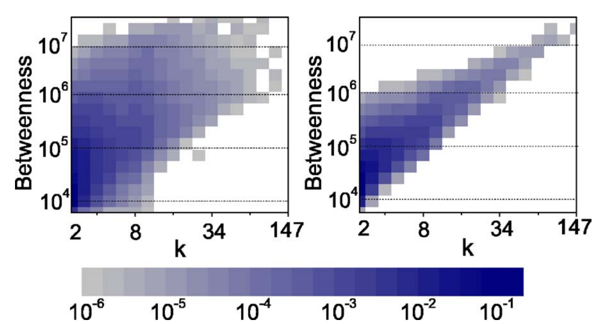

FIG. 7. (Color online) Log-binned probability density plots for the betweenness b and degree k of nodes. Data are the same as that in Fig. 6.

When an avalanche occurs, the fronts of the toppling nodes spread along geodesics, i.e., the shortest paths between nodes. Since the betweenness of a node is the number of the shortest paths passing through it, larger betweenness means that it will have a higher possibility to receive grains in avalanching processes. In the above sandpile model, the toppling threshold is the node's degree; thus, the node that has large betweenness but small degree will be easier to topple. As Fig. 7 shows, LESF networks have more such nodes than SFR networks, and the situation changes continuously for WLESF networks with increasing A. This could also account for the increase in larger avalanche events and the decrease of the avalanche area exponent τ as the network is more geographically constrained.

The cases with exponential degree distributions have also been studied, and the same geographical effect on avalanches has been observed. Thus, the geographical connections are seen to have a great influence on cascade events both for networks with a broad degree distribution, e.g., large scale-free networks, and for those with a narrower range of degrees, e.g., the networks with exponential degree distributions.

In conclusion, by the study of avalanching processes on geographical SF networks, we find that besides the reservoir effects of the hubs in SF networks, geography has great influences on the critical exponents of these systems. The same geographical effect also exists in networks with an exponential degree distribution. When the network is more geographically constrained, i.e., with heavier local connections, the avalanche area exponent τ will be smaller, which means more huge avalanche events. This implies a high risk for networks with heavier local connections to break down through cascading failures because they have a much higher vulnerability to huge avalanche events, due to the presence of denser connections, the larger number of smaller order cycles, and the larger betweenness of nodes with small degrees. Since many real networks that carry some kinds of loads—e.g., power, traffic, data packets—are imbedded in the 2D global surface and highly clustered, our results indicate that they will be at a higher risk to suffer breakdowns when there are node failures.

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